Lecture 32

P, NP

Two Sides of Computation

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Possibility

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Impossibility

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Possibility: Algorithms

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Deals with creating efficient algorithms for computational problems.

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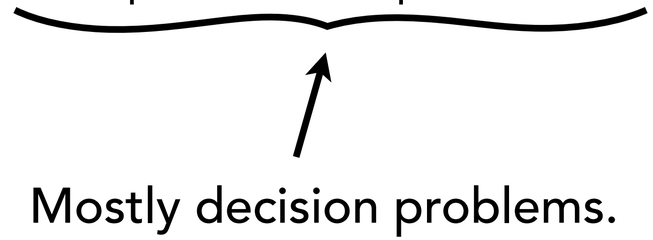

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Note: We will assume that in the problems an input is always provided in encoded binary form.

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Decision problems can be expressed as sets.

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The polytime algorithm you have in your mind is actually an exponential time algorithm.

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Note: **NP** stands for nondeterministic polynomial time.